Name: Solutions Class #:

Math 48A, Lesson 3: Graph More Popular Functions

5. GRAPH CUBIC FUNCTION (TYPE OF POWER FUNCTION)

Consider the quadratic function

$$f(x) = x^3$$

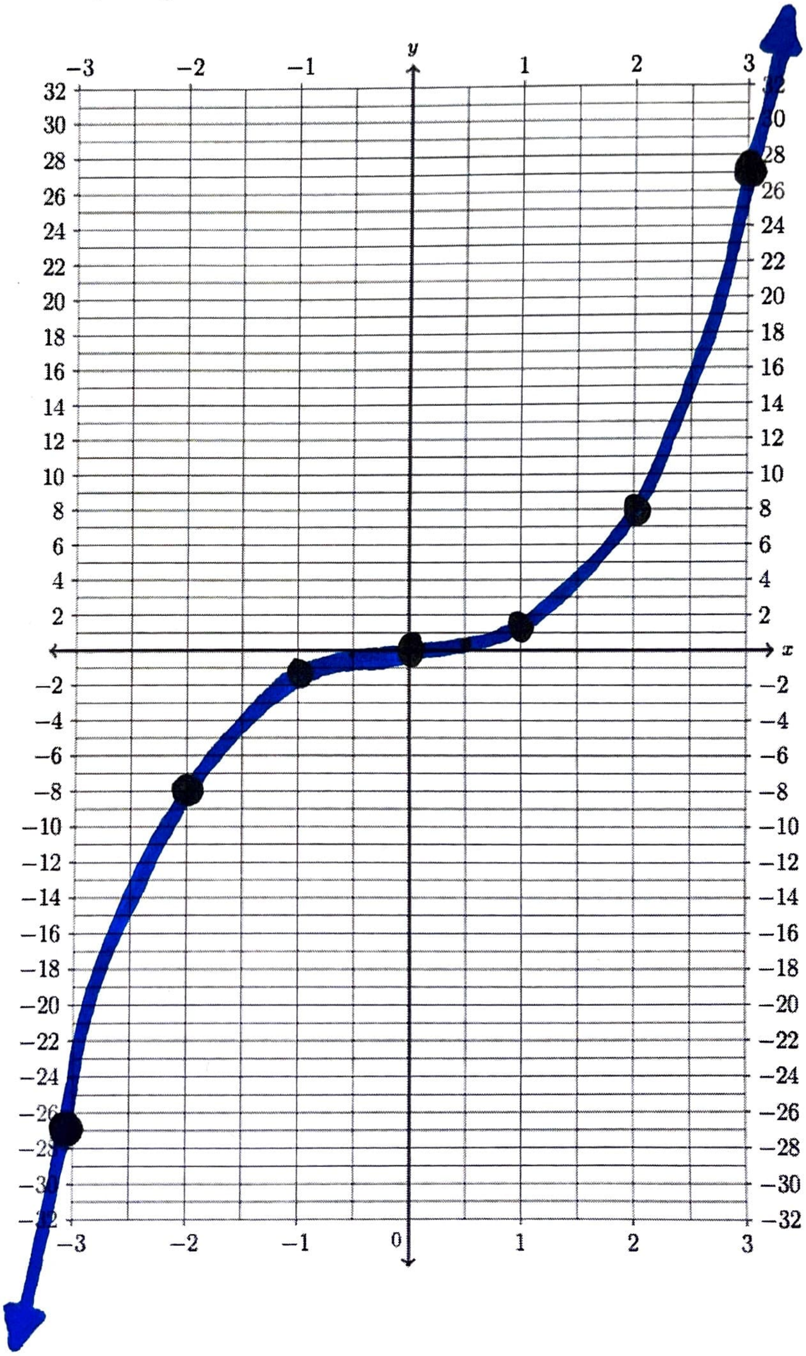
Fill out the table below. Then use that table to graph the quadratic function.

A. Fill in the table below

B. Plot these points on the axis provided

C. Interpolate between the points you plotted to create the graph of this function

Input	*Output values*
x	$f(x) = x^3$
-4	-64
-3	-27
-2	-8
-1	-1
$-\frac{1}{2}$	$-1/8$
0	0
$\frac{1}{2}$	$1/8$
1	1
2	8
3	27
4	64



See pages 2-3 of these solutions to learn more about how to get table values by hand. For other points, I used a TI calculator or mental math :)

Let's sample some points on graph of

$$f(x) = x^3$$

using pen and paper analysis:

Let $x = -3 \Rightarrow f(x) = f(-3)$

$$\Rightarrow f(-3) = (-3)^3$$

$$\Rightarrow f(-3) = (-3)\cdot(-3)\cdot(-3)$$

$$\Rightarrow f(-3) = \boxed{-27}$$

Let $x = \frac{-1}{2} \Rightarrow f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3$

$$= -\frac{1}{2} \cdot -\frac{1}{2} \cdot \frac{-1}{2}$$

$$= \frac{-1 \cdot -1 \cdot -1}{2 \cdot 2 \cdot 2} = \boxed{\frac{-1}{8}}$$

$$x = 0 \Rightarrow f(x) = f(0) = 0^3 = \underbrace{0 \cdot 0 \cdot 0}_{\text{three times}}$$

* powers counts the number times we multiply

$$x = -4 \Rightarrow f(x) = f(-4)$$

$$= (-4)^3$$

$$= (-4) \cdot (-4) \cdot (-4)$$

$$= 16 \cdot (-4)$$

	+	-
+	+	-
-	-	+

$$= -16 \cdot 4$$

$$= -(10+6)4$$

$$= -(40+24) = -64$$

Name:______________________ Class #:______

5D. What is the x–intercept of the cubic function $f(x) = x^3$?
(Write about how the x-intercept shows up in your graph from parts 5A – 5C).

x-intercept: • where the graph crosses the x-axis

• Since the x-axis is all points having y-coordinate $y=0$, notice that we can find x-intercept by setting

$$f(x) = 0$$

$$\Rightarrow f(x) = x^3 = 0 \Leftrightarrow \sqrt[3]{x^3} = \sqrt[3]{0}$$

$$\Leftrightarrow x = 0 \Leftrightarrow \boxed{(0,0) \text{ is unique x-intercept}}$$

5E. What is the y–intercept of the cubic function $f(x) = x^3$?
(Write about how the y-intercept shows up in your graph from parts 5A – 5C).

y-intercept: • where the graph crosses the vertical y-axis

• Since y-axis is all points having x-coordinate $x=0$, we find y-intercept by setting $x=0$:

$$f(x) = f(0) = 0^3 = 0$$

$\Rightarrow$ y-intercept for the graph of $f(x) = x^3$ is at the point $(0,0)$.

Name:________________________ Class #:______

5F. What is the domain of the cubic function $f(x) = x^3$?
(Write about how the domain shows up in your graph from parts 5A – 5C).

The doman of function $f(x)$ is all valid input points for the function $f(x)$.

In the case of function $f(x) = x^3$, we see we can evaluate x^3 at all real-valued inputs x. Thus we say the domain of $f(x)$ is all real numbers and we write $\text{Dom}(f) = \text{Dom}(x^3) = \mathbb{R} = \{\text{all real numbers}\}$

5G. What is the range of the cubic function $f(x) = x^3$?
(Write about how the range shows up in your graph from parts 5A – 5C).

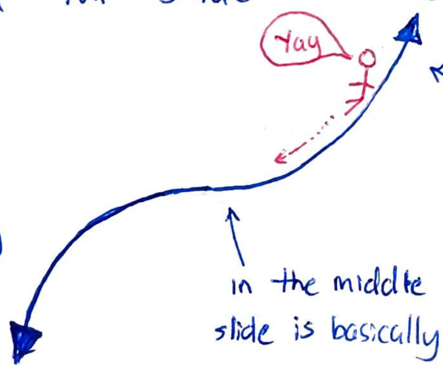
The range of a function $f(x)$ is the set of all outputs that the function $f(x)$ "achieves" as we evaluate $f(x)$ at every single input value in the domain of $f(x)$.

For the function $f(x) = x^3$, we see that the output values of x^3 go toward positive infinity as input values of x are positive and large. Similarly, as input values of x tend more and more negative, the output x^3 descends to negative infinity. In other words

$\text{Rng}(f) = \mathbb{R}$
all real numbers

5H. What does the graph of the cubic function $f(x) = x^3$ look like?

The graph of our cubic polynomial $f(x) = x^3$ looks like a fun slide to me



Toward right (as $x>0$) slide is very steep

toward left (as $x<0$) slide is very steep

in the middle (at $x=0$) slide is basically "horizontal"

Name:______________________ Class #:______

6. RATIONAL FUNCTION

Consider the absolute value function

$$f(x) = \frac{1}{x}$$

Fill out the table below. Then use that table to graph the absolute value function.

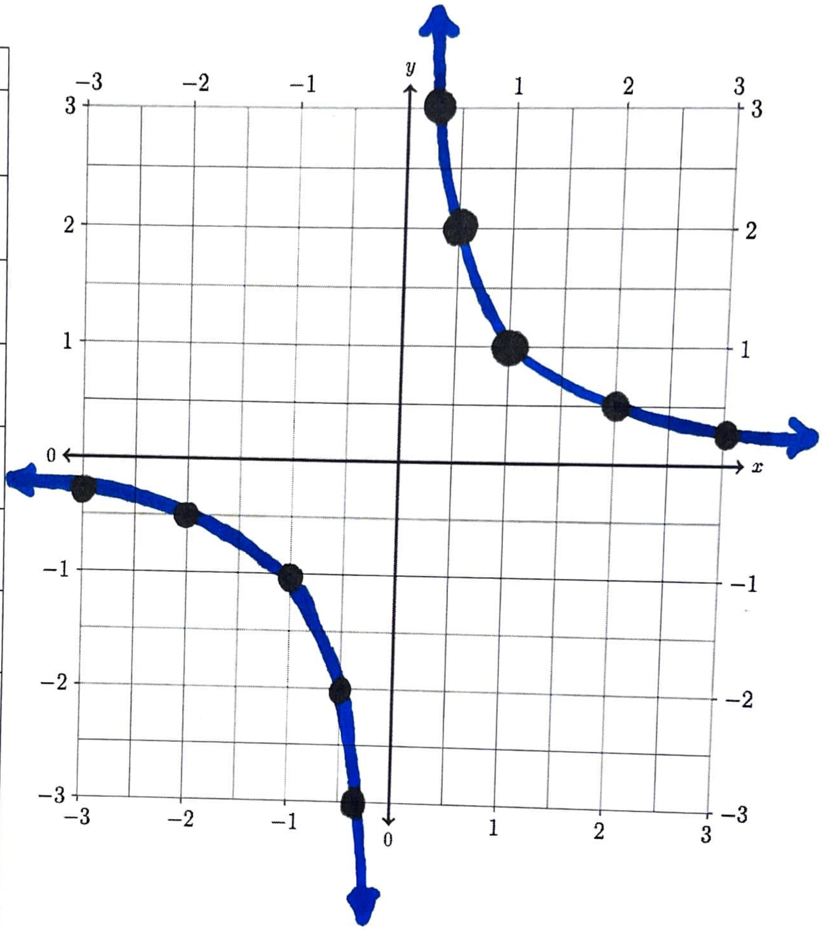
A. Fill in the table below

B. Plot these points on the axis provided

C. Interpolate between the points you plotted to create the graph of this function

Input	*Output*
x	$f(x) = \frac{1}{x}$
-3	$-1/3$
-2	$-1/2$
-1	$-1/1 = -1$
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
0	NA
$\frac{1}{3}$	3
$\frac{1}{2}$	2
1	$1/1 = 1$
2	$1/2$
3	$1/3$

see video w/ Jayden



Let $x = -3$ and find

$$f(x) = \frac{1}{x}\Bigg|_{x=-3}$$

$$\Rightarrow f(-3) = \frac{1}{-3} = -\frac{1}{3}$$

Let $x = -\frac{1}{2}$ and find

Numerator

division line

denominator

$$f\left(-\frac{1}{2}\right) = \frac{1}{-1/2}$$

$$= \boxed{1} \div \boxed{-\frac{1}{2}}$$

dividend

division symbol

divisor

PAGE 7

$$= \frac{1}{1} \div -\frac{1}{2}$$

Recall:

Reciprocal

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \boxed{\frac{D}{C}}$$

$$= \frac{1}{1} \cdot \frac{-2}{1}$$

$$\frac{A}{B} \cdot \frac{D}{C} = \frac{A \cdot D}{B \cdot C} = \frac{AD}{BC}$$

$$= \frac{1 \cdot -2}{1 \cdot 1}$$

$$= \frac{-2}{1}$$

$$f(-1/2) = -2$$

Let $x = 1/3$ and find

$$f(1/3) = \frac{1}{\frac{1}{3}} = 1 \div \frac{1}{3}$$

$$= \frac{1}{1} \div \frac{1}{3}$$

$$= \frac{1}{1} \cdot \frac{3}{1}$$

$$= \frac{1 \cdot 3}{1 \cdot 1}$$

$$= \frac{3}{1}$$

$$= 3$$

$\Rightarrow$ If $x = \frac{1}{3}$ and $f(x) = \frac{1}{x}$ then

$$\boxed{f\left(\frac{1}{3}\right) = \frac{1}{1/3} = 3}$$

Name:________________________ Class #:______

7. RATIONAL FUNCTION

Consider the absolute value function

$$f(x) = \frac{1}{x^2} = x^{-2} = \frac{1}{x} \cdot \frac{1}{x}$$

Fill out the table below. Then use that table to graph the absolute value function.

A. Fill in the table below

B. Plot these points on the axis provided

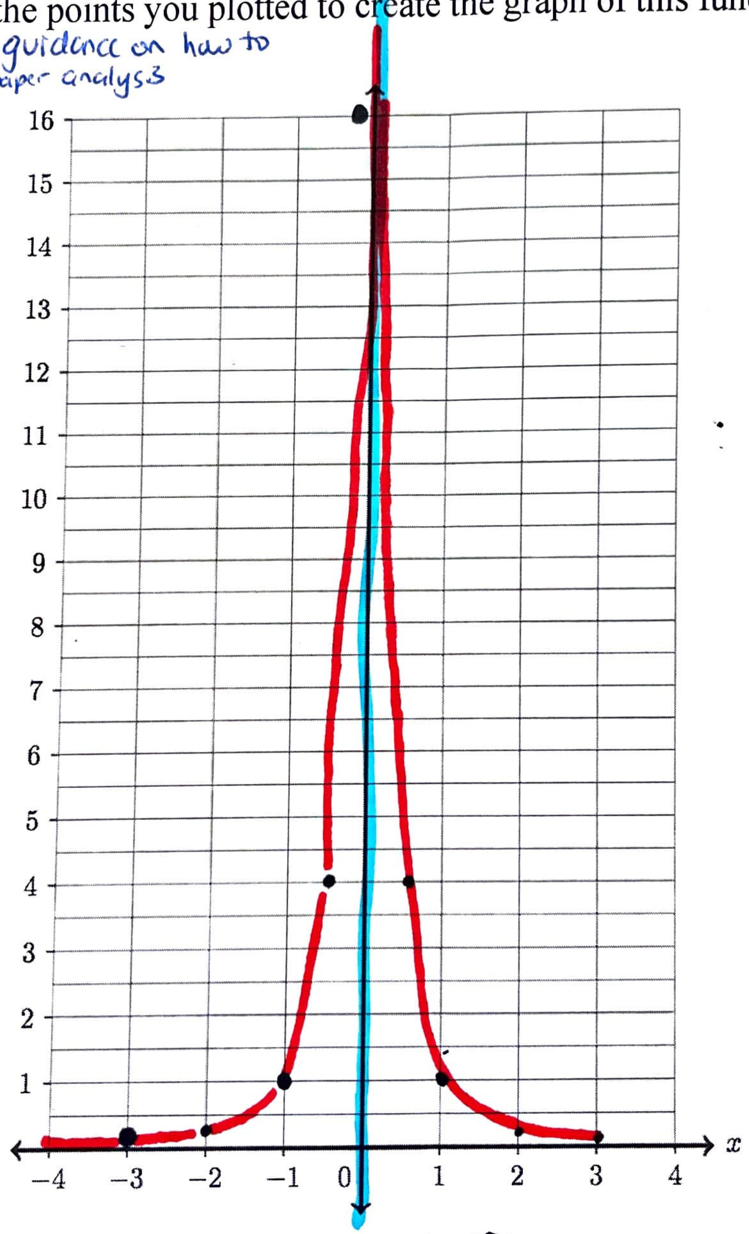
C. Interpolate between the points you plotted to create the graph of this function

See next pages for more guidance on how to get these using pen-and-paper analysis

Input	*Output*
x	$f(x) = \frac{1}{x^2}$
-3	$\frac{1}{9}$
-2	$1/4$
-1	1
$-\frac{1}{2}$	4
$-\frac{1}{4}$	16
0	NA
$\frac{1}{3}$	9
$\frac{1}{2}$	4
1	$1/1$
2	$1/4$
3	$1/9$

4 $1/16$

5 $1/25$



☐ Never cross x-axis

☐ Symmetric around y-axis

☐

$\boxed{x = -3}$

$$f(x) = \frac{1}{x^2}$$

$$\Rightarrow f(x) = f(-3) = \frac{1}{(-3)^2} = \frac{1}{9}$$

Note: $(-3)^2 = (-3)\cdot(-3)$
$= 9$

$\boxed{x = -2}$ $f(x) = f(-2) = \frac{1}{4}$

$$= \frac{1}{(-2)^2}$$

$$= \frac{1}{4} = 0.25$$

$x = \frac{-1}{2} \Rightarrow f\left(\frac{-1}{2}\right) = 4$

$$f(x) = f\left(-\frac{1}{2}\right) = \frac{1}{\left(\frac{-1}{2}\right)^2}$$

$$= 1 \div \left(\frac{-1}{2}\right)^2$$

P
E
MD
AS

Note: $\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$

$$\left[\frac{-1}{2}\right]^2 = \frac{-1}{2} \cdot \frac{-1}{2}$$

$$= \frac{-1 \cdot -1}{2 \cdot 2}$$

$$= \frac{1}{4}$$

$$= \boxed{1} \div \frac{1}{4}$$

Note: $1 = \frac{1}{1}$

$$= \boxed{\frac{1}{1}} \div \frac{1}{4}$$

Note:

$$\frac{A}{B} \div \boxed{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C}$$

Reciprocal

$$= \frac{1}{1} \cdot \frac{4}{1}$$

$$= \frac{1 \cdot 4}{1 \cdot 1}$$

$$= \frac{4}{1} = 4$$

division by 1
doesnt do anything

Name:________________ Class #:______

7D. What is the x–intercept of the rational function $f(x) = \frac{1}{x^2}$?
(Write about how the x-intercept shows up in your graph from parts 7A – 7C).

Notice: $\frac{1}{x^2}$ never crosses the x-axis

$\Rightarrow$ x-intercepts is the related to horizontal/straight across line (x-axis)

the point where graph touches x-axis

$\frac{1}{x^2}$ will never touch x-axis

$\Rightarrow$ $\boxed{\text{No x-intercepts}}$

7E. What is the y–intercept of the rational function $f(x) = \frac{1}{x^2}$?
(Write about how the y-intercept shows up in your graph from parts 7A – 7C).

- y-intercept is where the graph (or curve) crosses the straight up down line (y-axis)
- the graph of $\frac{1}{x^2}$ just keep going up or proceeds up and up and up and never crosses y-axis (touches)

we can't divide by zero $\frac{1}{0^2} = \frac{1}{0}$ ↙ undefined

- NO y-intercepts :

15

$$\frac{1}{x^2} \neq -1$$ output

$$\Rightarrow x^2 \cdot \frac{1}{x^2} \neq x^2 \cdot -1$$

$$\Rightarrow +1 \neq -1 \cdot \boxed{x^2}$$

- always $x \cdot x = x^2$
- Note: $2 \cdot x = x + x$
- never negative
- either zero or positive

This graph will never dip lower than x-axis :

$$\frac{1 \leftarrow \text{positive}}{x^2 \leftarrow \text{positive for } x \neq 0}$$

all outputs will be positive

Name:________________ Class #:______

7F. What is the domain of the rational function $f(x) = \frac{1}{x^2}$?
(Write about how the domain shows up in your graph from parts 7A – 7C).

- the domain is all x-values used in a function... the numbers that work
- domain $\frac{1}{x}$ is all valid inputs (x-values)
- dom $\left(\frac{1}{x}\right)$ is all real numbers except zero (we say can't divide by zero)

7G. What is the range of the rational function $f(x) = \frac{1}{x^2}$?
(Write about how the range shows up in your graph from parts 7A – 7C).

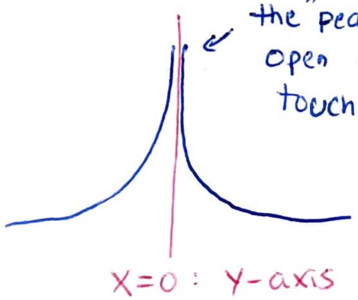
- the range is all valid outputs
- the range is all real outputs gotten by y-values

range $\left(\frac{1}{x}\right) = (0, \infty)$ or all positive real numbers (just keeps going up)

doesn't touch zero

7H. What does the graph of $f(x) = \frac{1}{x^2}$ look like?

The graph of $f(x) = \frac{1}{x^2}$ looks like an infinitely tall and slender volcano



Name:______________________ Class #:______

8. EXPONENTIAL FUNCTION

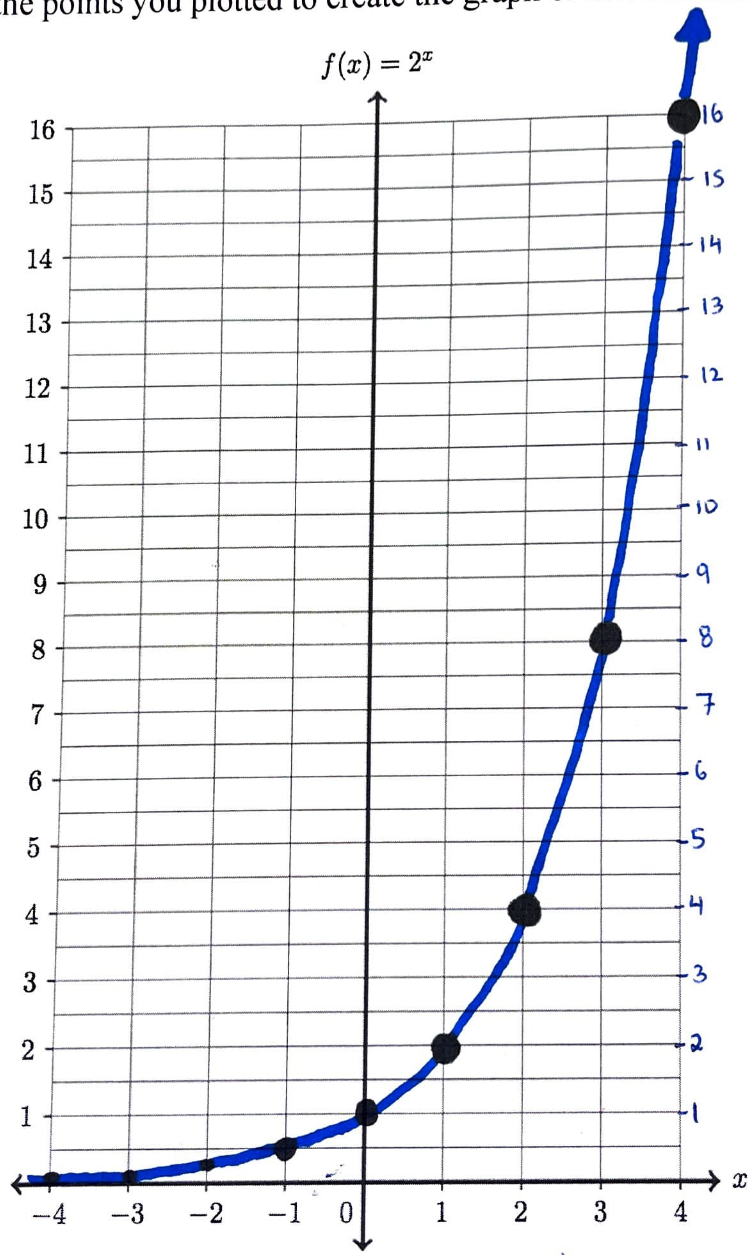
Consider the quadratic function

$$f(x) = 2^x$$

Fill out the table below. Then use that table to graph the quadratic function.

A. Fill in the table below
B. Plot these points on the axis provided
C. Interpolate between the points you plotted to create the graph of this function

x	$f(x) = 2^x$
−4	1/16
−3	1/8
−2	1/4
−1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32
6	64



* to move down from one row to the next, we multiply previous output by two

* to move up a row, divide last output by 2.

Let $f(x) = \boxed{2}^{\boxed{x}}$ ← variable exponent

constant base

- "f of x equals two to the exponent x"
- the function f(x) has a constant base two and a variable exponent value given by input x (this is an exponential function)

Let $x = 2$ and consider

$$f(2) = 2^x \Big|_{x=2}$$

↑ evaluation bar

$$= 2^2$$

$$= 2 \cdot 2 = 4 \Rightarrow \boxed{f(2) = 4}$$

Let $x = 3$ and find

$$f(3) = 2^x \Big|_{x=3}$$

"two to the exponent x evaluated at x equals three"

$$= 2^3 = 2^2 \cdot 2^1$$

$$= 2 \cdot 2 \cdot 2 = 8 = 4 \cdot 2$$

input ↘ output ↙

$$\Rightarrow \boxed{f(3) = 2^3 = 8} \Leftrightarrow (3, 8)$$

ordered pair

Let $x = 4$ and find

$$f(4) = 2^x \Big|_{x=4}$$

$$= 2^4 = 2^{3+1} = 2^3 \cdot 2^1$$

$$= \boxed{2 \cdot 2 \cdot 2} \cdot \boxed{2} = 2^3 \cdot 2^1$$

$$= 8 \cdot 2$$

$$= 16$$

left coordinate:

input
x-value

right coordinate
output
y-value

$$\Rightarrow \boxed{f(4) = 2^4 = 16} \Leftrightarrow (4, 16)$$

ordered pair

Let $x = 5$ and evaluate

$$f(5) = 2^x \Big|_{x=5}$$

$$= 2^5 = 2^{4+1} = 2^4 \cdot 2^1$$

$$= \boxed{2 \cdot 2 \cdot 2 \cdot 2} \cdot 2 = 2^4 \cdot 2^1$$

$$= 16 \cdot 2$$

$= 32$

$\Leftrightarrow \boxed{f(5) = 2^5 = 32} \Leftrightarrow (5, 32)$

1st coordinate

2nd coordinate

ordered pair

What patterns do I see?

- Each time we take the next integer value of x, we take the output of the previous integer x-value and multiply by 2

Notice: $2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2} = \frac{1}{2^1}$

$2^{-2} = \frac{1}{2} \div 2$

reciprocal

$= \frac{1}{2} \div \frac{2}{1}$: $\frac{A}{B} \div \boxed{\frac{C}{D}} = \frac{A}{B} \cdot \boxed{\frac{D}{C}}$

$= \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1 \cdot 1}{2 \cdot 2} = \frac{1}{2^2}$

$= \frac{1}{4}$