Name : ____________________ Class Number: _____

ENGR 11: Mortgage Loan Calculator Problem (A-Level Task)

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1. **How Loans Work** Partial Sums in Action

   As working professionals, we are expected to be financially independent adults. Part of the goal of earning a paycheck is to secure a steady income stream to cover our day-to-day expenses. However, there are some special purchases that may necessitate the use of borrowed money to afford an expensive good or service for which we can only pay a fraction of the price upfront.

   To make up the difference between the downpayment and the purchase price, we may may choose to take out a loan. In such a financial agreement, we borrow money from a *lender*. As the **lessee** (the person who has borrowed money), we recognize this loan as a *liability* (a sum of money that we must repay). This type of financial arrangement exists for student loans, car loans, home mortgages, business loans, and credit cards loans. In this problem, we will write a MATLAB `.m` script file to calculate key variables associated with the loan payment process. Hopefully, the work we do on this problem will come in helpful if if we ever find ourselves considering a particular loan. One of the major goals we might have as a borrower is to finish the process of paying off a loan, called **amortization**, as quickly and strategically as possible.

   Usually, when planning to repay a loan, it is important to consider the initial principal borrowed, the specified annual interest rate offered on the loan, and the period of time desired to pay off the loan. Let's create a model to describe the amortization process. We begin with the following variables

$$
\begin{aligned}
P_0 &= \text{the INITIAL PRINCIPAL (term amount) to be repaid,}\\
P_n &= \text{the OUTSTANDING PRINCIPLE (principle remaining to be paid)}\\
&\quad\ \text{at the end of the } n\text{th interest period,}\\
M &= \text{the minimum periodic payment}\\
N &= \text{the total number of PAYMENT PERIODS during the life of the loan}\\
m &= \text{be the NUMBER OF PERIODIC PAYMENTS PER YEAR,}\\
i^{(m)} &= \text{the ANNUAL INTEREST RATE, expressed as a decimal,}\\
i &= \text{the INTEREST RATE PER INTEREST PERIOD}
\end{aligned}
$$

   where $i = i^{(m)}/m$. The table below represents the payment process during the first 6 periods.

Amortization Schedule

Period	Monthly Payment	Interest Paid	Principal Paid	Outstanding Principal
0				P_0
1	M	iP_0	$M - iP_0$	$P_0 - (M - iP_0) = P_1$
2	M	iP_1	$M - iP_1$	$P_1 - (M - iP_1) = P_2$
3	M	iP_2	$M - iP_2$	$P_2 - (M - iP_2) = P_3$
4	M	iP_3	$M - iP_3$	$P_3 - (M - iP_3) = P_4$
5	M	iP_4	$M - iP_4$	$P_4 - (M - iP_4) = P_5$
6	M	iP_5	$M - iP_5$	$P_5 - (M - iP_5) = P_6$

In this problem, we are going to use the descriptions and table above to derive (from first principles) closed formulas for both P_n and M. Then, we will write a script file to calculate these values.

A. To begin, please use the recursive definition for P_n illustrated in the table above to show that

$$P_n = \left(P_0 - \frac{M}{i}\right)(1+i)^n + \frac{M}{i}.$$

B. Now, let's derive the formula for the minimum periodic payment. To do so, please assume we repay the loan exactly at the end of the Nth payment period, meaning that we can set $P_N = 0$. Use this assumption to show that the **minimum periodic payment** M is given by the formula

$$M = \frac{iP_0}{1-(1+i)^{-N}}.$$

C. Write a simple script file that calculates both the outstanding principle P_n due at the very beginning of the $(n+1)-$st interest period and the minimum periodic payment M due at the end of each interest period. In this script file, assign values for the variables P_0, $i^{(m)}, m$, and N. Then, using these variables, calculate both P_n and M using the formulas for these values given below.

$$P_n = \left(P_0 - \frac{M}{i}\right)(1+i)^n + \frac{M}{i} \qquad \text{and} \qquad M = \frac{iP_0}{1-(1+i)^{-N}}.$$

D. Please include all relevant features I mentioned in Problem (2.C.iii) above.

E. Once you have your script file working, please visit the Nerd Wallet Online Loan Calculator. Enter the following loan amounts online:

$$P_0 = \$500\,000, \qquad i^{(m)} = 4.25\%, \qquad N = 30 \text{ years}$$

Use the online calculator to calculate the values of M. Now, use your script file to do the same. What do you notice?