Name : ____________________ © Jeffrey A. Anderson **Class Number**: ______

Math 1C: Calculus III

In-Class Exam 2, Version 9A

Tuesday 11/12/2019: 6pm - 8:15pm

How long is this exam?

- This exam is scheduled for a 135 minute period.
- Make sure you have 5 sheets of paper (10 pages front and back) including this cover page.
- There are a total of 8 separate questions on this exam including:
  - 8 Free-response questions (with NO subproblems) (50 points)
  - 1 Optional, extra credit challenge problem (5 points)

How will your written work be graded on these questions?

- Your work should show evidence of original thought and deep understanding. Work that too closely resembles the ideas presented in Jeff's lesson notes will likely NOT earn top scores. Work that does not demonstrate individualized, nuanced understanding will likely NOT earn top scores.
- Read the directions carefully. Your work will be graded based on what you are being asked to do.
- In order to earn a top score, please show all your work. In most cases, a correct answer with no supporting work will NOT earn top scores. What you write down and how you write it are the most important means of getting a good score on this exam.
- Neatness and organization are IMPORTANT! Do your best to make your work easy to read.
- You will be graded on accurate use of the notation we studied in this class.

What can you use on this exam?

- You may use less-than or equal to six double-sided or 12 single-sided note sheets.
- Each note sheet is to be no larger then 11-inches by 8.5-inches (standard U.S. letter-sized paper).
- You may write on both sides of your note sheets.
- You must write your own note sheet and these sheets must be handwritten. You are NOT allowed to use a friends notes.
- PLEASE SUBMIT ALL OF YOUR NOTE SHEETS WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

What other rules govern your participation during this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note sheets), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

For problems 1 - 4, let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable function with explicit representation $z = f(x, y)$. Let $A\big(a, b, f(a, b)\big)$ be a point on the surface

$$S_f = \{(x, y, z) \,:\, (x, y) \in D \text{ and } z = f(x, y)\}.$$

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ be a unit vector in the domain of function f.

1. (6 points) Please derive the limit definition of the directional derivative from first principles. If you're confused where to start, please follow the 5 steps process to constructing a derivative that we discussed in our Lesson 11 videos.

2. (4 points) Using the limit definition for the directional derivative of f in the direction of $\mathbf{u}$ at the point (a, b) that you derived in problem 1 above, show how to construct a composite function $g(t)$. This single variable function should have the property that the derivative $g'(t)$ is the same value as the limit we constructed to compute the directional derivative in problem 1.

3. (4 points) Derive the dot product formula for the directional derivative. Be sure to specifically refer to the the function $g(t)$ from problem 2 above along with the multivariable chain rule with two intermediate variables and one independent variables. When appropriate, please explicitly state and use the multivariable chain rule in your work. Also, make sure to explain the value of t that you use to take the ordinary derivative in this derivation.

4. (6 points) Using your work in problem 3, explain which unit vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ in the domain D give

   A. the direction of steepest ascent on the surface.

   B. the direction of no change on the surface.

   C. the direction of steepest descent on the surface.

   Please provide evidence that your concept images associated with these directions incorporate multiple categories of knowledge including verbal, graphical, and symbolic representations of these ideas. To earn top scores, your solution should combine the work you did in problem 3 with the cosine formula for the dot product. Also, please make specific connections to between your explanations of each direction and your knowledge of the extreme values of the cosine function.

For problems 5 - 6, let $f(x,y) = 15 - x^2 - 4y^2 + 2x - 40y$.

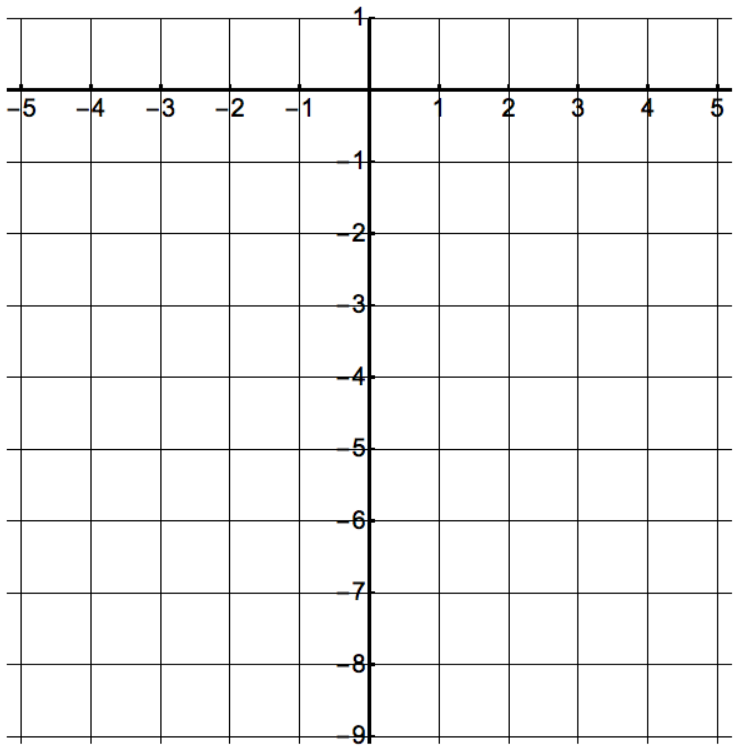
5. (8 points) Find a vector-valued equation for the tangent line to the level curve

$$L_{100}(f) = \{(x,y) \,:\, f(x,y) = 100\}$$

at the point $(-3,-5)$.

6. (6 points) On the axes below, sketch the level curve $L_{100}(f)$ and it's the tangent line from problem 5 above. Also, sketch the vector $\mathbf{u} \in \mathbb{R}^2$ with tail at point $(-3, -5)$ where $\mathbf{u}$ is the unit vector in the direction of the gradient vector $\nabla f(-3, -5)$ given by

$$\mathbf{u} = \frac{\nabla f(-3, -5)}{\|\nabla f(-3, -5)\|_2}$$



Now, use full sentences to explain how your graph above relates your knowledge about the shape of the surface $f(x, y)$ and your solution to problem 6 above.

For problems 7, 8, and 9, choose two out of these three problems you want me to grade for your first attempt during our in-class exam session. For the problem you would like to skip grading for your in-class attempt, please mark a big "X" through that problem. For the problem you skip, you can submit your solutions in your exam corrections. For now, focus on the two of these problems that you feel most comfortable with and give your best effort.

7. (8 points) Among all the points on the graph of $z = 10 - x^2 - y^2$ that lie above the plane $x + 2y + 3z = 0$, find the point farthest from the plane.

8. (8 points) Consider the function

$$f(x,y) = (x-1)^2 + (y-2)^2$$

Find the minimum and maximum values of $f(x,y)$ subject to the constraint that $x^2 + y^2 = 45$.

9. (8 points) Consider two multivariable functions defined by implicit equations

$$F(x, y, z) = x^2 + y^2 - 2 = 0 \qquad \text{and} \qquad G(x, y, z) = x + z - 4 = 0.$$

Notice that $F(x, y, z) = 0$ defines a cylinder while $G(x, y, z) = 0$ defines a plane. The intersection of these two surfaces forms an ellipse E. Find the parametric equation for the line tangent to E at the point $P_0(1, 1, 3)$.