Lesson 10: Chain Rule Handout
Reference: Brigg's "Calculus: Early Transcendentals, Second Edition"
Topics: Section 12.5: The Chain Rul, p. 907 - 916

Theorem 12.7. *p. 908* ***Chain Rule (Two Intermediate and One Independent Variable)***

Let $z = z(x, y)$ be a differentiable function of x and y on its domain, where $x = x(t)$ and $y = y(t)$ are differentiable functions of t on an interval I. Then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Note: A useful way to remember this theorem is to use the *tree diagram*:

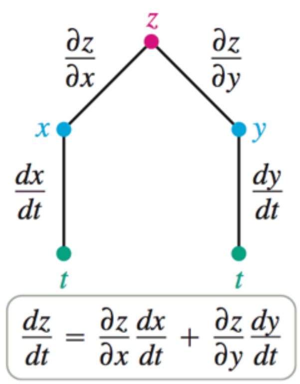

Figure 12.54

Theorem. *p. 908* ***Chain Rule (Three Intermediate and One Independent Variable)***

Let $w = w(x, y, z)$ be a differentiable function of x, y and z on its domain, where $x = x(t)$, $y = y(t)$ and $z = z(t)$ are differentiable functions of t on an interval I. Then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

Note: A useful way to remember this theorem is to use the *tree diagram*:

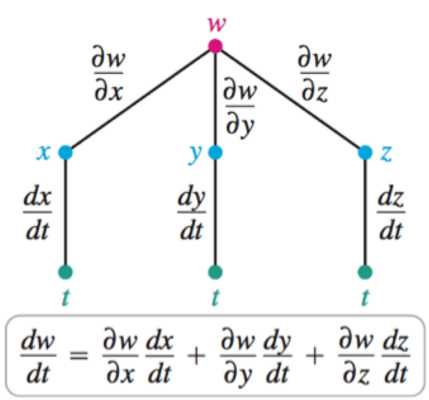

Figure 12.55

Theorem 12.8. *p. 909* ***Chain Rule (Two Intermediate and Two Independent Variable)***

Let $z = z(x, y)$ be a differentiable function of x and y on its domain, where $x = x(s, t)$ and $y = y(s, t)$ are differentiable functions of s and t. Then

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}, \qquad \text{and} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t},$$

Note: A useful way to remember this theorem is to use the *tree diagram*:

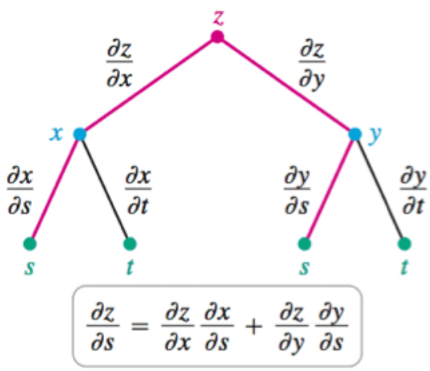

Figure 12.57

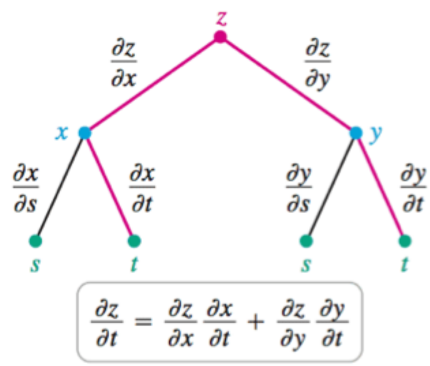

Figure 12.58

Theorem. *p. 909* ***Chain Rule (Three Intermediate and Two Independent Variable)***

For this situation, we can draw the following *tree diagram*:

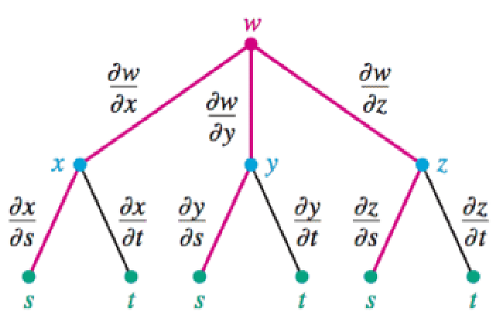

Figure 12.59

Theorem 12.9. *p. 911* ***Implicit Differentiation***

Let F be differentiable on its domain and suppose that $F(x, y) = 0$ defines y as a function of x. Provided that $F_y \neq 0$, then

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$