Name : ____________ Math 1C, Lesson 5: In-Class Problems **Class Number**: ______

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1. Consider the following two functions:

$$\mathbf{r}_2(t) = \langle x(t), y(t) \rangle \qquad \text{and} \qquad \mathbf{r}_3(t) = \langle x(t), y(t), z(t) \rangle$$

   a. Draw a function map diagram in the form $f : D \longrightarrow C$ for each of the functions above. Specifically identify the domain space and the codomain of each function.

   b. Explain why the functions $\mathbf{r}_2(t)$ and $\mathbf{r}_3(t)$ are known as vector-valued functions.

   c. Explain the similarities and differences between multivariable real-valued functions and vector-valued functions.

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2. Define two lines $L_1(t)$ and $L_2(s)$ intersect at a single point in $\mathbb{R}^3$, where

$$\mathbf{L}_1(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 1+t \\ 2t \\ -1+3t \end{bmatrix} \qquad \text{and} \qquad \mathbf{L}_2(s) = \begin{bmatrix} x(s) \\ y(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} 3+2s \\ 1+s \\ -2-s \end{bmatrix}$$

   Find the point (x, y, z) of intersection. Then, find the angle θ between the lines.

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3. Find the equation for the plane the contains the curve

$$\mathbf{r}(t) = t\mathbf{i} + \frac{1}{2}t^2\mathbf{k}.$$

   Graph this curve using Mathematica to confirm your hypothesis.

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4. Find the vector-valued function equation of the line the joins the points $P(0, 0, 0)$ and $Q(8, 7, 2)$. Now, restrict the domain of your function to produce an equation for the line segment between these two points. Use this work to graph this line segment and the two points in Mathematica.

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5. Determine the equation of the line $\mathbf{r}(\tau)$ that is perpendicular to the following two lines

$$L_1(t) = \langle\, 7t,\ 1+3t,\ 4t \,\rangle \qquad \text{and} \qquad L_2(s) = \langle\, 1+s,\ -14+3s,\ -20+4s \,\rangle$$

   and passes through the point of intersection of the lines $L_1(t)$ and $L_2(s)$. Graph all three lines on the same axes in Mathematica.

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6. Find the point (if it exists) at which the plane $y = -5$ intersects with the line

$$\mathbf{r}(t) = \langle\, 4t+1,\ -1+4t,\ t-6 \,\rangle\,.$$

   If the point of intersection exists, graph the plane, the line and the point of intersection on the same axes in Mathematica.

7. Consider the implicit (relation) equation for a circle in $\mathbb{R}^2$ given by

$$(x-h)^2 + (y-k)^2 = r^2$$

Use this as a starting point to derive the vector-valued equation for a circle in $\mathbb{R}^2$ with radius r and center at points (h,k). Draw a diagram that explains how you converted one the scalar equation in cartesian coordinates into a vector equation in polar coordinates.

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8. Sketch the graph of the following ellipses by hand. Specifically identify the center point and the length of each semi-axis. After you have sketched your graph by hand, graph each of these ellipses in Mathematica to check your work.

   A. $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$

   B. $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$

   C. $12x^2 + 5y^2 = 60$

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9. Challenge equation: Recall the implicit (relation) equation for an ellipse in $\mathbb{R}^2$ given by

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Derive this equation from first principles.