Lesson 7 : Graphs and Level Curves.

similar to

Example 12.2.1 p. 874 Let's find the domain and range of

$$g(x,y) = \sqrt{9 - x^2 - y^2}$$

Top half of sphere :

Radius $r = 3$

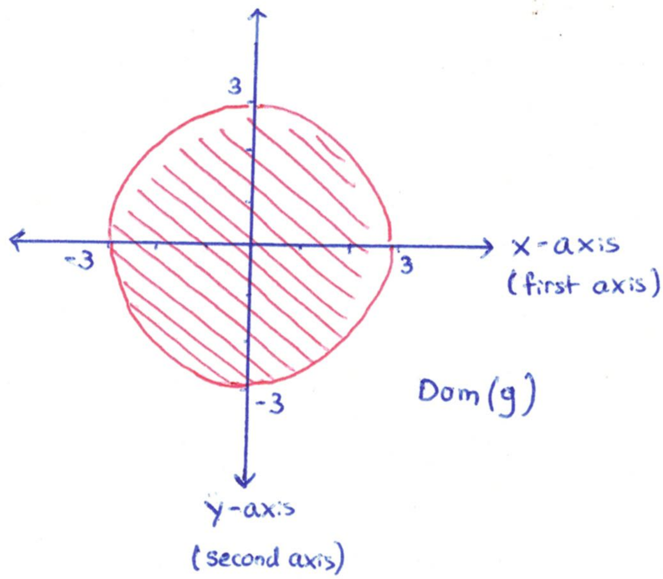
Solution : We Know for the square root function,

the input must be greater than are equal to zero

$$D = \text{Dom}(g) = \{ (x,y) \in \mathbb{R}^2 : 9 - x^2 - y^2 \geq 0 \}$$

$$= \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9 \}$$

$\Rightarrow$ The domain of $g(x,y)$ is the disk centered at $(0,0)$ and radius 3



Example 12.2.1 p. 874 ...

To find the $\text{Rng}(g(x,y))$, we recall

$$\text{Rng}(g) = \{ z \in \mathbb{R} : z = \sqrt{9 - x^2 - y^2},\ (x,y) \in D \}$$

We know that the square root function never drops below 0

$$\Rightarrow \quad z = \sqrt{9 - x^2 - y^2} \geq 0$$

Notice also that $0 \leq x^2 + y^2 \leq 9$

$$\Rightarrow \quad 0 \geq -x^2 - y^2 \geq -9$$

$$\Rightarrow \quad 9 \geq 9 - x^2 - y^2 \geq 0$$

$$\Rightarrow \quad 3 \geq \sqrt{9 - x^2 - y^2} \geq 0$$

$$\Rightarrow \quad 3 \geq z \geq 0$$

$$\Rightarrow \quad \text{Rng}(g) = \{ z : 0 \leq z \leq 3 \} = [0, 3].$$

Example 12.2.1 p.874...

To graph $g(x,y) = \sqrt{9 - x^2 - y^2}$, we use our Knowledge of spheres, mathematica and algebra to find our graph

Example: Enhanced via mathematica

$$z = \sqrt{9 - x^2 - y^2}$$

$$\Rightarrow z^2 = 9 - x^2 - y^2$$

$$\Rightarrow x^2 + y^2 + z^2 = 9$$

$$\Rightarrow x^2 + y^2 + z^2 = 3^2$$

$\Rightarrow$ $g(x,y)$ is a half sphere centered at $(0,0)$ with radius $r = 3$.

See mathematica notebook associated with Lesson 7 for more details

Another method for visualizing functions $f: \mathbb{R}^2 \to \mathbb{R}$ is called a contour map, Major idea is to draw maps on which points with constant "elevation" are joined to form contour lines or level curves.

Def: Level curves of $f(x,y) \cancel{= k}$ are curves with equations

$$f(x,y) = k$$

for $k \in Rng(f) \subseteq \mathbb{R}$.

Let's sketch the level curves of

$$g(x,y) = \sqrt{9 - x^2 - y^2} \quad \text{for} \quad k = 0, 1, 2, 3$$

To do so, we systematically set our outputs to a constant k and find the curves in x and y ~~to~~ that correspond to this situation

$$h = \sqrt{9 - x^2 - y^2}$$

$$\Rightarrow h^2 = 9 - x^2 - y^2$$

$$\Rightarrow x^2 + y^2 = 9 - h^2$$

See mathematica for enhanced graphics of contour lines

Similar to

Exercise 12.2.17 p. 882)

[Enhanced using Mathematica]

Let $f(x,y) = \ln\left(9 - 9x^2 - y^2\right)$.

Let's find the domain and range of $f(x,y)$.

Solution: Recall from Math 1A, $z = \ln(w) \iff e^z = w$

$$\Rightarrow w > 0$$

$$\Rightarrow 9 - 9x^2 - y^2 > 0$$

$$\Rightarrow 9 > 9x^2 + y^2$$

$$\Rightarrow 9 > [3x]^2 + y^2$$

We can find this region by some analysis on ~~equation~~ inequality.

$$3x^2 + y^2 < 9$$

This is an equation for an ellipse

Along x-axis $(y=0)$: $3x^2 + 0^2 < 9 \Rightarrow x^2 < 3$

$$\Rightarrow -\sqrt{3} < x < \sqrt{3}$$

Along y-axis $(x=0)$ $3\cdot 0^2 + y^2 < 9 \Rightarrow y^2 < 9$

$$\Rightarrow -3 < y < 3$$

Exercise 12.2.41 p. 883

← see mathematica notebook