Name:______________________ Class #:______

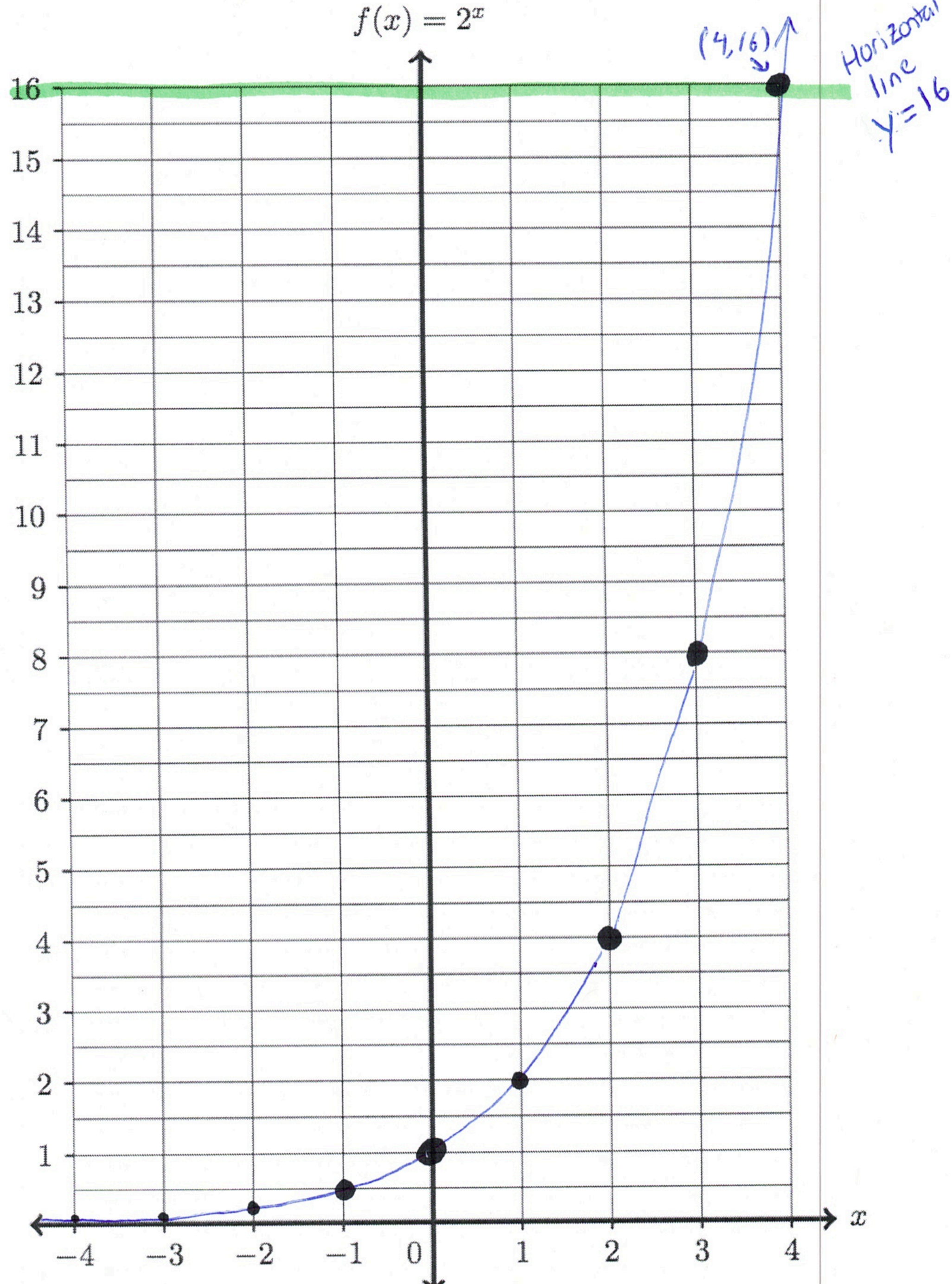
4. VISUALIZE A FORWARD PROBLEMS USING A GRAPH

4A. Consider the function

$$f(x) = 2^x = y$$

Fill out the table below to graph the forward problem for this function.

Input x	Output $f(x) = 2^x$
-4	$\frac{1}{16}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32
6	64



- To move from one row to the one below, we multiply last entry by 2
- To move from one row to the one above we divide the last enty by 2

Let $f(x) = 2^{x} = y$

Let's solve the forward problem many

given input

$$2^{\boxed{x}} = \boxed{y}$$

predefined (structure of problem)

unknown output

Example: $x = 2 \Rightarrow f(x) = 2^{x}$

$$\Rightarrow f(2) = 2^{2}$$

$$\Rightarrow f(2) = 2 \cdot 2$$

$$\Rightarrow f(2) = 4$$

Example : $\boxed{x = 3} \Rightarrow \boxed{f(x) = 2^x}$

$\Rightarrow \boxed{f(3) = 2^3}$

$\Rightarrow \boxed{f(3) = 2 \cdot 2 \cdot 2}$

$\Rightarrow f(3) = 8$

Note : $2^3 = (2 \cdot 2) \cdot 2$

$= 2^2 \cdot 2^1$

$= 2^2 \cdot 2 = 4 \cdot 2$

$$\boxed{\text{Remember: } x^m \cdot x^n = x^{m+n}}$$

Let's find some patterns...

$$2^{0} = 1 = \frac{2^{1}}{2} = \frac{2}{2} = \frac{2^{1}}{2^{1}} = 2^{1-1}$$

$$2^{-1} = 1 \div 2 = \frac{1}{2}$$

$$2^{-2} = \frac{1}{2} \div \frac{2}{1}$$

$$= \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{4}$$

flip

- $\frac{A}{B} \div \boxed{\frac{C}{D}} = \frac{A}{B} \cdot \boxed{\frac{D}{C}}$
- $\frac{A}{B} \cdot \frac{D}{C} = \frac{A \cdot D}{B \cdot C}$

Name:______________________ Class #:_____

4B. In your own, simple language, explain why the work you did in Problem 4A represents a forward problem.

We see that in Problem 4A, we solved a forward problem because

$\boxed{\text{Forward Problem}}$

given input

$$f(\boxed{x}) = 2^{\boxed{x}} = \boxed{Y}$$

desired output

- Forward problems start with given input to a pre-defined function and end with the desired output
- Backwards problem start with a given out put to a pre defined function and ends with the desired input to function that produce those outputs

Name:______________________ Class #:_____

4C. Look back at Problem 4A. Make a conjecture (a mathematical guess) about what the backward problem for the function $f(x) = 2^x = y$ would look like. Write the symbols and verbal description so that you describe this using both words and mathematical notation.

desired input

Backwards Problem

$f(\boxed{x}) = \boxed{y}$

given output

Start with a given output

and ends with the

desired input needed

to produce that output

Note: to swap between forward & backward problem, we pretty much just flip inputs and outputs

Name:______________________ Class #:______

5. VISUALIZE A BACKWARD PROBLEMS USING A GRAPH

5A. Consider the function

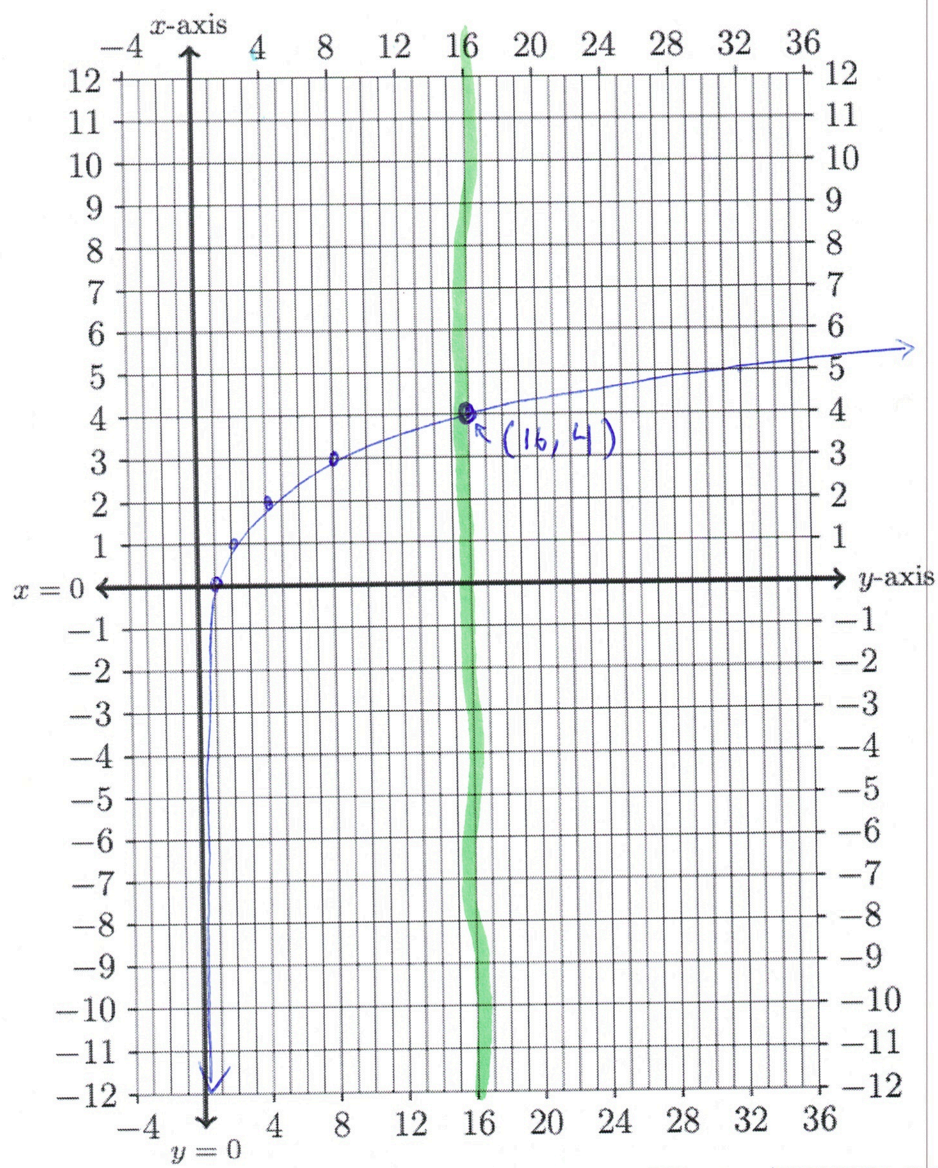
$$f(x) = 2^x = y$$

Fill out the table below to graph the backward problem for this function.

input output

$y = 2^x$	x
$\frac{1}{16}$	-4
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3
16	4
32	5
64	6

vertical line
x=16



☐ This inverse relation is a function since it passes the vertical line test

* swap x and y : inverse exponential (logarthm)

Name:________________________ Class #:______

5B. In your own, simple language, explain why the work you did in Problem 5A represents a backward problem.

- Remember in the forward problem

$$f(\boxed{x}) = 2^{\boxed{x}} = \boxed{y}$$

we start with the $\boxed{\text{given input } x}$

for the pre-defined exponential function

$f(x) = 2^x$. Then we calculate the desired

output $2^x = y$ (we work forward

starts here $\longrightarrow$ works forward

from input to output.

- In backward problem, we swap the direction...

we work backwards from $\boxed{\text{given output}}$ $\boxed{y}$

and find the $\boxed{\text{desired input needed}}$ for our

pre-defined function so that

$$f(\boxed{x}) = 2^{\boxed{x}} = \boxed{y}$$

works backwards $\longleftarrow$ start here

Think about this:

given input

Forward Problem

$$2^{\boxed{x}} = \boxed{y}$$

desired output

Exponential Function evaluation

↑

inverse

↓

desired output

Backward Problem

$$2^{\boxed{y}} = \boxed{x}$$

given input

inverse algebraic equation

- ☐ In order to transform a forward problem to a backward problem we switch the $\boxed{y}$ and $\boxed{x}$
- ☐ to find an inverse function we switch the $\boxed{x}$ and $\boxed{y}$

Name:______________________ Class #:______

5C. Look back at Problem 4 and 5. What is the relationship between the forward and backward problems for the function $f(x) = 2^x = y$. Write your response in both symbols and verbal description so that you describe this using both words and mathematical notation.

start here work towards here

In the forward problem $2^x = y$

the input is the exponent on base 2

and we then calculate the output value y

that results when we raise 2 to input x.

In the backward problem, the roles of input

and output swap (we change x to y and vice versa)

Specifically, we ask for given value x

work towards here

Start here

$$2^y = x$$

what exponent do we need on base 2 so that

2 to that exponent y equals x.

Name:______________________________ Class #:______

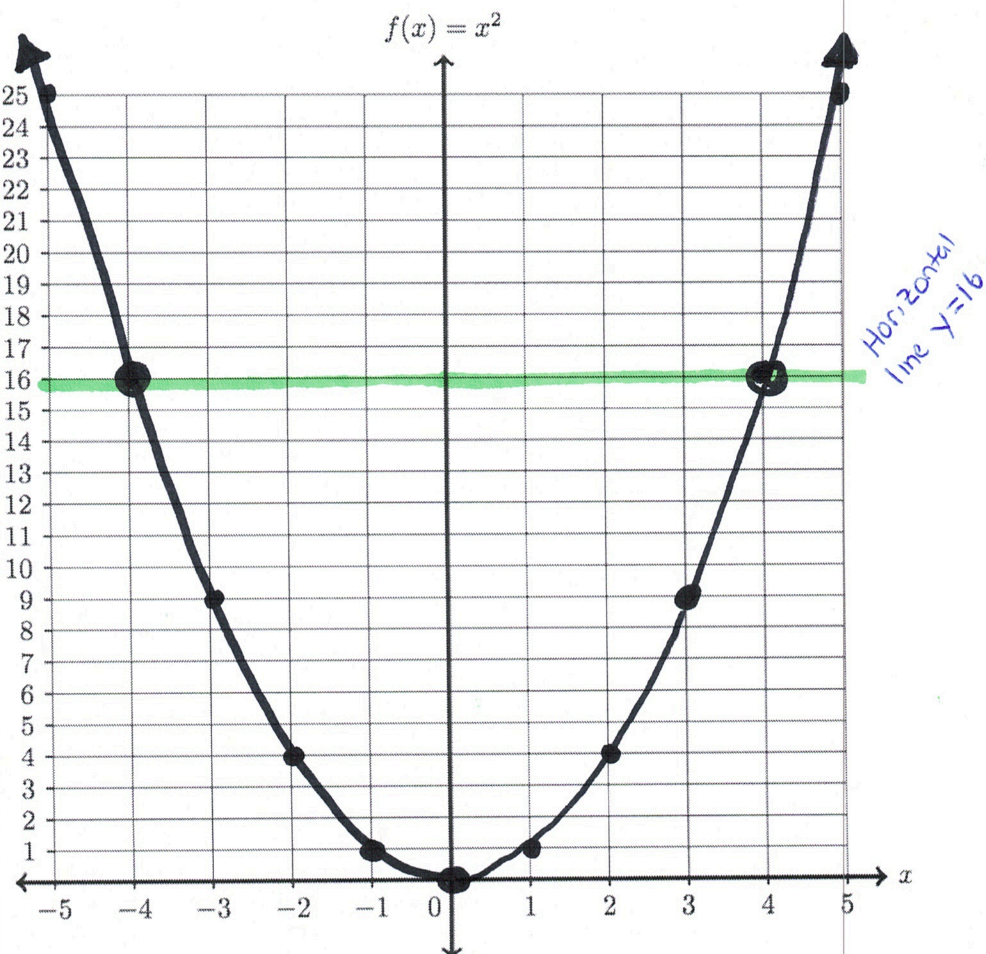
6. VISUALIZE A FORWARD PROBLEMS USING A GRAPH

6A. Consider the following function

$$f(x) = x^2 = y$$

Create a table of values and graph the resulting curve on these axes below.

Input	*Output*
x	y
−5	25
−4	16
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9
4	16
5	25



(−4, 16) and (4, 16)

To move from foward to backwards we flip inputs to outputs

Name:_______________________ Class #:______

6B. In your own, simple language, explain why the work you did in Problem 6A represents a forward problem.

In this work, we start with a given input value x for our predefined function $y = f(x)$ and then calculate desired output value y

$$f(x) = x^2 = y$$

start here

work towards here

6C. Look back at Problem 6A. Make a conjecture (a mathematical guess) about what the backward problem for the function $f(x) = x^2 = y$ would look like. Write the symbols and verbal description so that you describe this using both words and mathematical notation.

In the backward problem, we flip directions

work backwards towards x

$$f(x) = x^2 = y$$

Start here

In other words, the role of input and output switch.

Name:________________________ Class #:______

7. VISUALIZE A BACKWARD PROBLEMS USING A GRAPH

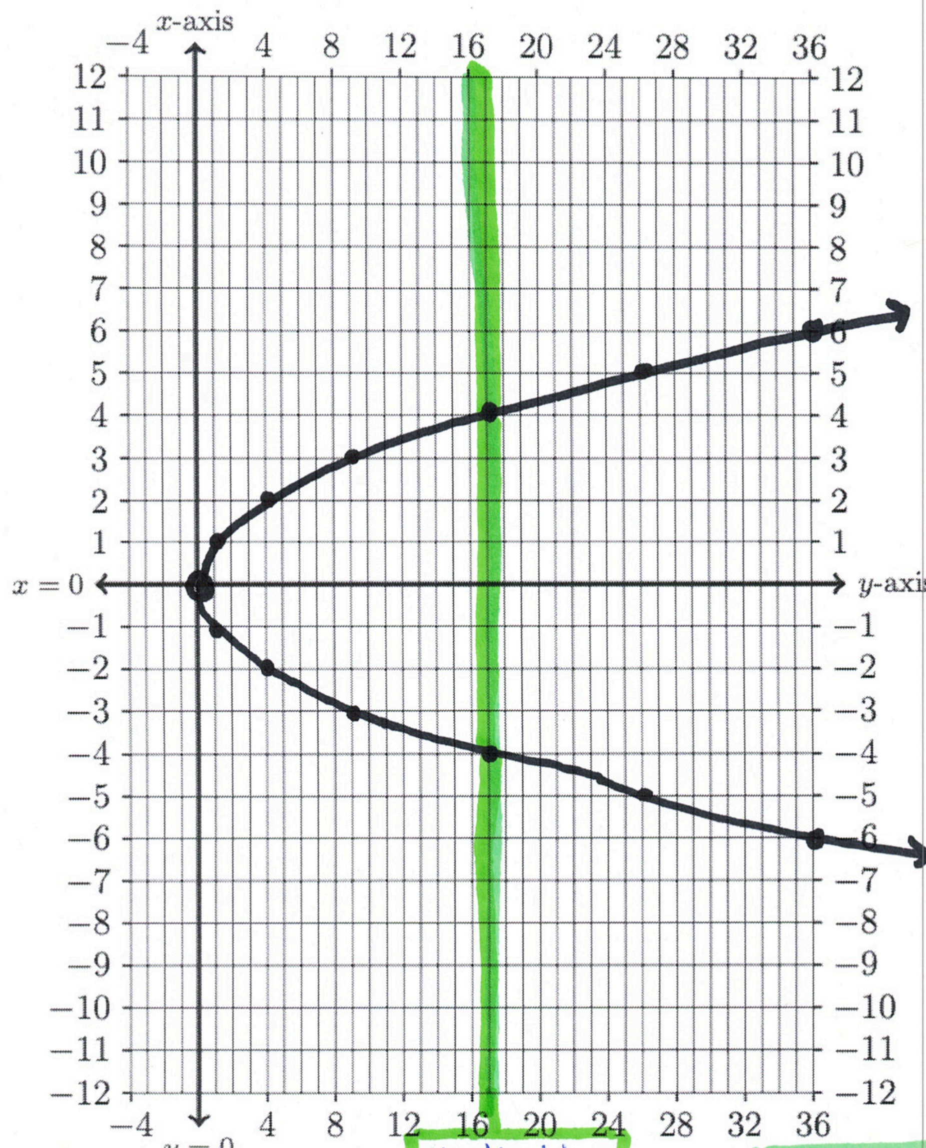
7A. Consider the function

$$f(x) = x^2 = y$$

Fill out the table below to graph the backward problem for this function.

inverse relation NOT function
(fails vertical line test)

$y = x^2$	x
−1	NONE
0	0
1	1 or −1
4	2 or −2
9	± 3
16	± 4
25	± 5
36	± 6
49	± 7
64	± 8
81	± 9



vertical line
x = 16

vertical

(16, 4)
and
(16, −4)

(input , output)

horizontal