Quick reference guide to the linear-algebraic nodal analysis (LANA) algorithm

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INPUT:

Suppose we are given a complete description of an electric circuit containing only resistors, dc voltage sources, and dc current sources. The LANA algorithm proceeds with the steps provided below.

ALGORITHM:

1. Identify and label all circuit nodes
2. Model the circuit as a directed graph
3. Create all circuit matrices
   - 3A. Create the entire incidence matrix
   - 3B. Create the node voltage potential vector
   - 3C. Create the voltage-drop vector
   - 3D. Create the current vector
4. State the entire set of circuit equations
   - 4A. Kirchhoff's current laws (KCLs)
   - 4B. Branch constitutive relations (BCRs)
   - 4C. Kirchhoff's voltage laws (KVLs)
   - 4D. Combine the circuit equations
5. Identify ordinary and generalized nodes
6. Create a minimal set of independent node potentials
   - 6A. Impose one constraint for each voltage source
   - 6B. Impose a single constraint for the ground node
   - 6C. Combine constraints together
7. State and solve the equilibrium equation for the circuit

OUTPUT:

This algorithm outputs modeled values for all independent node voltage potentials. We can use these node potential values to solve for any circuit variable we desire.

WHY IS THIS USEFUL?

LANA transforms electric circuits into mathematical models involving a nonsingular linear-systems problem in the form

$$A_r^T\, G\, A_r\, \mathbf{u} = A_r^T\, G\, \mathbf{b} - \mathbf{f}$$

where the matrix $K = A_r^T\, G\, A_r$ is almost always positive definite. This matrix structure is popular in STEM modeling contexts and has received much attention in the development of numerical linear algebra.
