

Design and Construction of an Eigenvalue Apparatus (Version 1)

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In this document, we demonstrate how to build an eigenvalue apparatus. This system includes a metal frame to support a spring-coupled pair of pendula. The design presented here is one of many possible realizations of the apparatus discussed in a corresponding paper titled *Make the eigenvalue problem resonate with our students*. The following descriptions are meant to be guidelines, not rigid prescriptions. The goal of this work is to build one or more of these for classroom use that encourages students and instructors to explore. If you are not comfortable with hand tools, engage a colleague from the physics or engineering department or a retired woodworker in your community to join you. Make the construction a team experience. There are many areas where new or alternate materials, assembly, and operation can be employed. Be creative and enjoy the process.

Step 1: Study the general design

The coupled pendulum system consists of a frame that supports two pendula in a way that each pendulum moves in one direction only, as shown in Figure 1. Each pendula is of sufficient length to allow oscillation slow enough to be measured by students with a video camera or smart phone. Any frame of suitable dimensions can be used. The frame presented in this document is made from a pair of commercial wireframe shelving structures (known as “Postal Shelves”), attached top to bottom. For more information about the specific make and model used here, please see the parts source list is at the end of this document. The system can be aligned to be vertical with four screw adjusted leveling feet. The system we chose allowed some degree of portability for movement to and from classrooms.

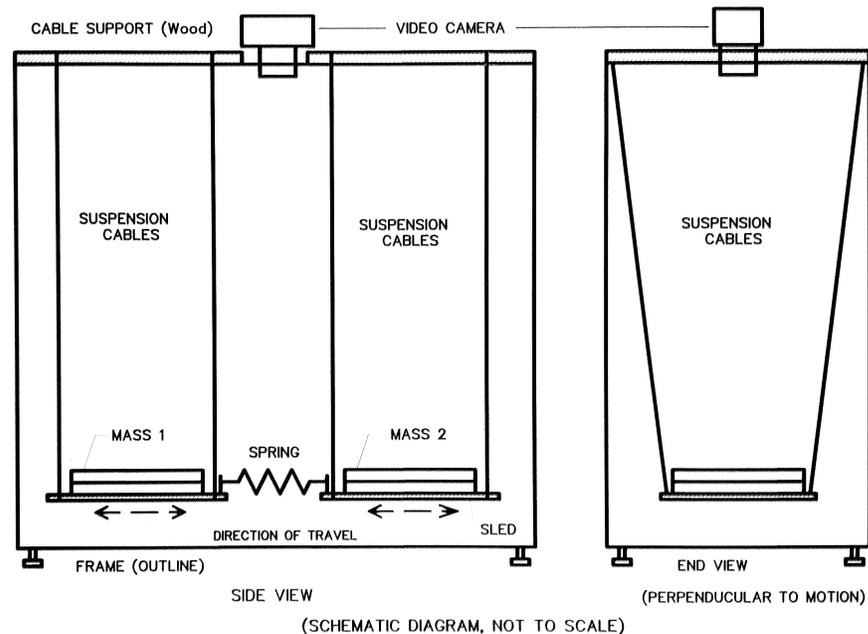


Figure 1: Schematic Diagram. This shows two views: the left side of this figure shows two pendula, each of which consists of a platform or sled that can support selectable masses. The two pendula are the same length and can be connected with a spring. The direction of motion is indicated with the dashed arrows below the platforms. The right side shows the end view of these. The end view shows the cable support at the top is wider than the cable attachments to the platforms. The upper shelf is plywood and is drilled so that a video camera can be used to capture the motion of the pendula from a top view.

Step 2: Assemble the metal frame

We assemble the wire-shelving units according to the manufacturer's instructions. To stack the shelving units, we remove end caps of the vertical rods and insert 5/8" wooden dowel between the upper frame and the lower frame. We use a hacksaw to remove some of the wire shelves in the top shelf to allow the video capture with a camera. We then attach pieces of plywood to the shelf. These are drilled and slotted to allow installation of the pendulum cables and drilled to allow the camera to view the motion, as seen in Figure 2 below.



Figure 2: Assembled Frame. This photo shows a “short version” of the pendulum frame prior to installing the pendula.

We also place a piece of fiberboard on the lowest shelf to conveniently hold measuring tools and to provide a uniform background for video images. We cover the board with blue paper. This photo is of an implementation that uses only one shelf system. This makes a pendulum length of 0.65 meters rather than 1.37 meters used in the media published in the *Make the eigenvalue problem resonate with our students* article. This is more portable than the original, taller system. The extra shelf seen in Figure 2 is not needed structurally, but shows how shelf wires can be cut away to allow travel or clearance of the camera at the top.

Step 3: Build the platforms for each pendulum cart

The pendulum support platforms, displayed in Figure 3, can be made from any material that is rigid enough to support the masses and the various spring attachments. We have used particleboard approximately 6 mm thick with dimensions 17.7 cm by 13.3 cm. None of these dimensions are critical. The platforms are drilled in four places to accommodate the support cables.

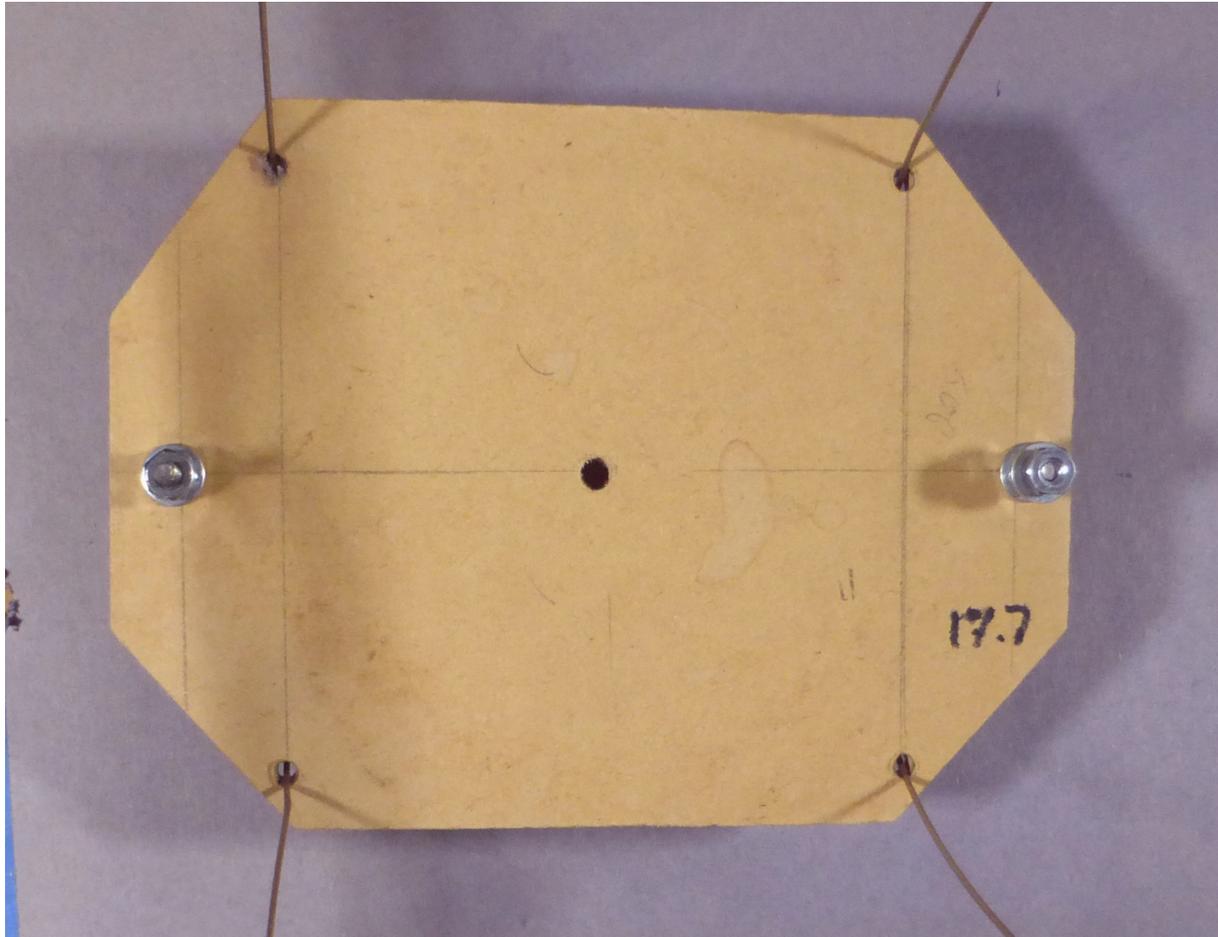


Figure 3: Platform for pendulum masses. This photo shows the platforms we attach to the top of our metal frame using wire cables. The pendulum masses sit on top of these platforms.

One useful design tip to keep in mind is to measure the masses you plan to use prior to cutting the pendulum platforms. Remember that each platform should be able to fit *both* the pendulum masses and extra screws that serve as attachment points for the spring. The platform pictured in Figure 3 above was cut in as a hexagon to allow all for enough space to fit all components on each cart.

Step 4: Create a mechanism to attach the spring to each platform

A simple machine screw and nut is attached to each sled. The two ends of a spring are clamped to each screw so the two pendula are coupled by a selectable spring. A variety of springs can be used for this purpose. The work reported here was done with a spring constant of ~ 3.47 kg/m. To learn more about how to measure the spring constant, please see the “Measuring the Spring Constant Handout (.pdf)” available on the support website for this paper. One possible supplier for springs is listed in the parts source list which can be found on the last few pages of this document.



Figure 4A: Attachment point for spring to platform made from a screw, nuts and washers.

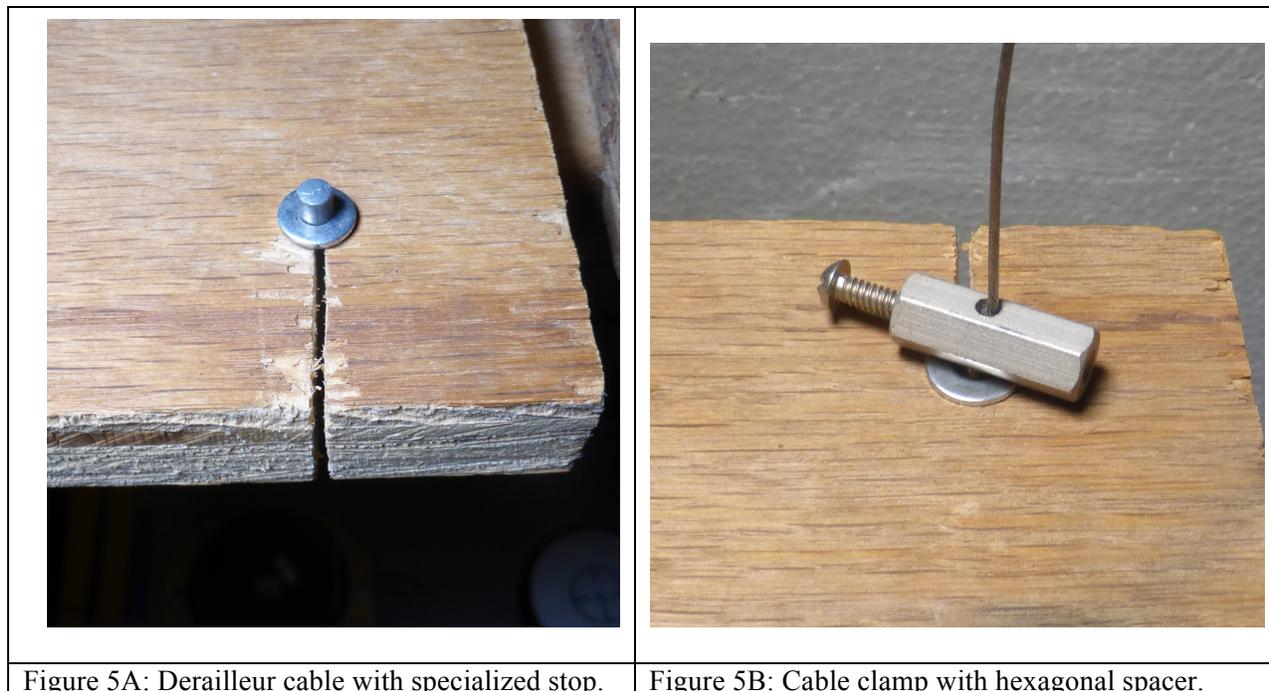


Figure 4B: Mechanism to attach spring to pendulum cart using the attachment point.

The spring is clamped to each platform with a machine screw drilled in the center of the interior end of the sled (in this case, we used a 8-32 by 1” screw, though the choice of size is not critical). The screw is bound to the platform with a nut. Then, another nut is added along with two washers and a third nut to very top of the screw, as seen in Figure 4A. The end loops of the springs are then be clamped between the washers, as shown in Figure 4B.

Step 5: Connect the pendulum carts to the metal frame using steel cables

The cables used are stranded steel cables that were sold in a bicycle shop as tandem bicycle derailleur cables. These have both strength and flexibility. A less expensive but quite satisfactory choice are stranded wire cables for picture hanging that are available in almost any hardware store. The cables are attached to the top looped through the slots in the plywood, as shown in Figure 4A. The cables can be clamped in several ways. The derailleur cables have a small sphere on one end that is larger than the slot. On the other we have used a drilled and tapped sleeve with a set screw. A simpler mechanism might be a simple wood screw with a washer installed in the plywood. The cable can be clamped by tightening the screw, seen in Figure 4B.

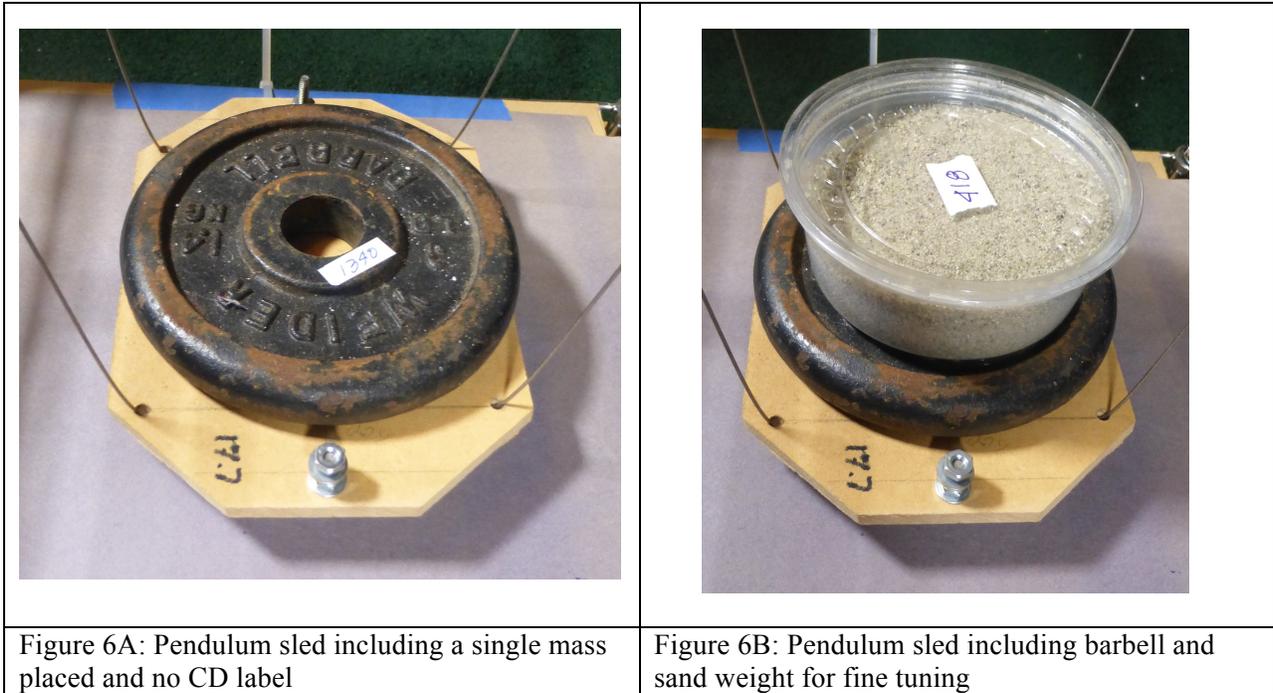


The mounting of the cables in the slots, which are made by first drilling a hole at the desired final position and then cutting a slot from the near edge to the hole, offers the benefit of ease of installation and removal. This facilitates easy transportation to and from classrooms and to other sites for demonstrations. The structure supports motion of the masses along a single axis since the width of the attachment points at the top, 38 cm, are wider than at the bottom where the hole separations are 10.7 cm. This is the same stability principle used in the familiar Newton's cradle:
https://en.wikipedia.org/wiki/Newton%27s_cradle.

Notice that the lower ends where the cables attach to the sleds are just wide enough to accommodate the masses. In contrast, the upper ends of the cables are attached to points on the frame that are approximately three times as wide as the lower ends. This design feature tends to suppress motion perpendicular to the preferred one-dimensional motion along a single axis.

Step 6: Place masses on support frames

For our first implementation the masses on each pendulum were two 3 lb. barbell weights. These have a diameter of 0.13 meters. Any items of similar mass can be employed. We measure the masses of the sleds, weights, and cables with an ordinary digital kitchen scale (Ozeri ZK14, with a capacity of 5 kg in 0.001 kg steps—see the parts source list at the end.). We add small masses, such as washers, to assure the mass equality of each pendulum. Cable strands support each pendulum. The strands are attached at the top of the frame with clamps. With four cables, the two sleds maintain level orientation while traveling in an arc. The system maintains small angle movement: the pendulum length, measured from the top pivot to the center of mass of the weights and sled, is 1.37 meters and the maximum arc length is typically 0.06 meters or less.



The barbell weights of 3 lbs. each may not have the “fine tuning” needed for some measurements. In particular, you may find it hard to design a system in which both pendulum sleds hold masses having identical measurements. For this purpose, we can use alternate masses. Figure 6B shows a mass of 0.418 kg that is made from a plastic container from a delicatessen filled with sand.

Step 7: Place masses on support frames

After we place the masses on the pendulum sleds, we mark the location of the center of mass of each sled is identified by using a white CD label with a black circular marking added, as shown in Figure 7. The main purpose of this feature is to enhance the accuracy and precision of image processing software in tracking the motion of the mass.



Figure 7: Pendulum mass marked with CD label to enhance image processing.

Step 8: Place a reference length calibration scale under the masses

The eigenvalue analysis utilizes a measurement of the motion amplitude and its velocity. This requires a reference length scale that is captured at the same time as the motion with the video camera. A reference length scale must be placed at the same level as the sleds to avoid parallax errors. Initially we installed a wooden bar with markings at separations of 0.1 meter, 0.2 meter etc. A length scale that is even more convenient than a separate scale is to note that the diameter of the white CD label used to mark the center of mass is 0.118 meters in diameter. This can be used by the measurement software as a length scale.

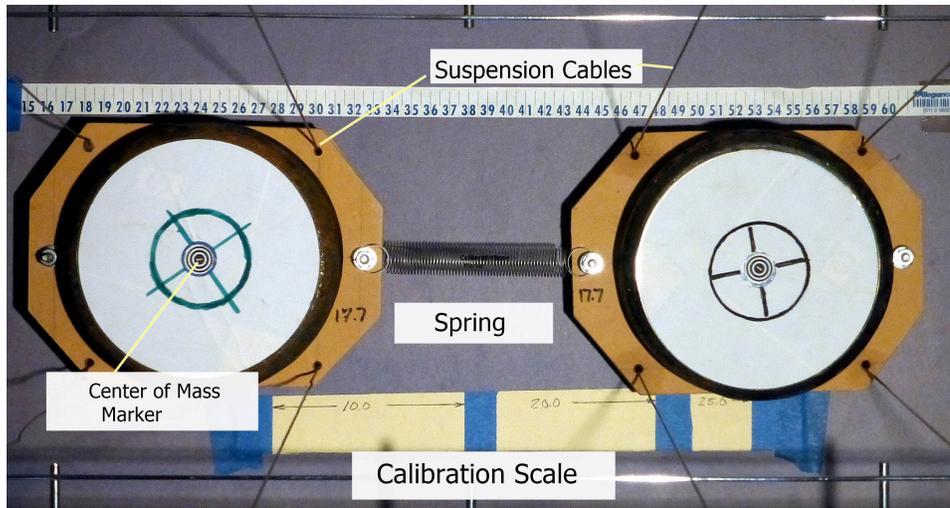


Figure 8: View from above of the two masses, the coupling spring, and the reference scales of the apparatus. The camera, located in the center of the top of the structure, looks down.

Step 9: Adjust the system to be vertical

Make a plum bob by attaching a small weight, such as a ½ inch nut to a piece of string. Hold this tool near each vertical corner of the framework. Adjust the length of each foot on the frame by turning the adjustable feet one way or the other until the all are vertical in two directions.

Step 10: Assure equal pendulum lengths

Each cable passes from the top of the structure through two holes in the platform and then back to an adjustable clamp in the top of the structure (see photos above). When the masses are added to the platforms each cable length must be adjusted so that the platforms are level, and more importantly, that the two pendula have the same length. A convenient method for this is to cut a piece of card stock so that it just fits between the bottom of one end of the first platform and the bottom surface. Then move this reference tool to the other end. Adjust the cable length with the upper clamp so that this end of the platform is the same as the first. Then repeat this process with the other platform. This must be done with the platforms fully loaded. An accurate check of length is to determine the period of each pendulum as it oscillates freely (i.e., no springs attached.) Make appropriate adjustments to fine-tune the length.

Step 11: Operate the system

In operation the pendula are set in motion by hand. The first mode, synchronous operation, can be launched simply by gently pushing the top of the frame in the direction of the “x-axis” the desired motion direction. Gently push in the same way that is done when a child is on a swing (a few millimeters of pushing motion that is repeated is all that is needed). The anti-synchronous operation is more challenging. Ideally the two masses must be separated by the same distance and then released at the same time. This can take several tries. Let the oscillations go for many seconds to allow perpendicular motion (in the “y” direction) to be suppressed. The mixed mode can be launched by holding one pendulum fixed at the lowest position, then moving the other a few centimeters to the outside and letting them both loose at the same time. As before, wait a minute or so to allow small y direction excursion to be suppressed.

We have recorded the motion of the pendula is then recorded with video capture with a simple digital camera with zoom lens and video capture (Panasonic DMC-Z525). The camera operates at 30 frames per second. The camera views the motion from the top of the frame so that the motion of the masses is parallel to the camera’s focal plane. It is VERY important that while capturing the video the camera or the operator does not move the structure, especially in the same direction as the pendulum motion.

The video is analyzed with “Tracker” a free motion and capture software application (For FOOTNOTE <http://physlets.org/tracker/>). A single frame image from one video is shown in the photograph below.

The analysis process in the software first sets the alignment axis in the direction of the pendulum motion. The dimensions are set by locating two marks on the reference length scale and recording them. Then one definable moving feature is selected, in this case the feature is the reference mark of the center of mass of one of the masses. The software then can automatically track the motion of that feature frame by frame and record the time and the locations of the feature as it moves. The time and locations are then displayed in a table (spread sheet format) that can be exported to a conventional spread sheet for subsequent analysis. The software calculates the velocity and acceleration for each mass using the time and location data. Position data is accurate so that experimental errors of less than a few percent are possible.

Parts Source List (including prices and links)

Shelving system for frame
[Home Depot Price \\$21.88](#)
(as of Feb 2018)



Digital Scale: Ozeri ZK14-S
[Google Express Price \\$16.75](#) This or similar products are available at numerous sites.



Hexagonal aluminum standoff for cable clamps from [Jameco Electronics: Price \\$0.61](#)



Parts Source List (including prices and links)

The springs used here are relatively weak compared with the spring selections available in hardware stores. We have been using those from [Pasco Scientific with Price \\$36.](#) (plus shipping).



A less expensive source for viable springs might be from [Eisco labs \(via Amazon\)](#)

